Mathematical Reasoning

1. Which of the following sentences are statements ? Justify
   - (i)   A triangle has three sides.
   - (ii)   0 is a complex number.
   - (iii) Sky is red.
   - (iv)   Every set is an infinite set.
   - (v) $15 + 8 > 23$.
   - (vi) $y + 9 = 7$.
   - (vii) Where is your bag?
   - (viii) Every square is a rectangle.
   - (ix)   Sum of opposite angles of a cyclic quadrilateral is 180°.
   - (x) $\sin^2 x + \cos^2 x = 0$

2. Find the component statements of the following compound statements.
   - (i)   Number 7 is prime and odd.
   - (ii)   Chennai is in India and is the capital of Tamil Nadu.
   - (iii) The number 100 is divisible by 3, 11 and 5.
   - (iv)   Chandigarh is the capital of Haryana and U.P.
   - (v) $\sqrt{7}$ is a rational number or an irrational number.
   - (vi)   0 is less than every positive integer and every negative integer.
   - (vii) Plants use sunlight, water and carbon dioxide for photosynthesis.
   - (viii) Two lines in a plane either intersect at one point or they are parallel.
   - (ix)   A rectangle is a quadrilateral or a 5 - sided polygon.

3. Write the component statements of the following compound statements and check whether the compound statement is true or false.
   - (i)   57 is divisible by 2 or 3.
   - (ii)   24 is a multiple of 4 and 6.
   - (iii) All living things have two eyes and two legs.
   - (iv)   2 is an even number and a prime number.

4. Write the negation of the following simple statements
   - (i)   The number 17 is prime.
   - (ii) $2 + 7 = 6$.

(iii)   Violets are blue.

(iv) $\sqrt{5}$ is a rational number.

(v)   2 is not a prime number.

(vi)   Every real number is an irrational number.

(vii)   Cow has four legs.

(viii)   A leap year has 366 days.

(ix)   All similar triangles are congruent.

(x)   Area of a circle is same as the perimeter of the circle.

5.   Translate the following statements into symbolic form

(i)   Rahul passed in Hindi and English.

(ii) x and y are even integers.

(iii)   2, 3 and 6 are factors of 12.

(iv)   Either x or $x + 1$ is an odd integer.

(v)   A number is either divisible by 2 or 3.

(vi)   Either $x = 2$ or $x = 3$ is a root of $3x^2 - x - 10 = 0$

(vii)   Students can take Hindi or English as an optional paper.

6.   Write down the negation of following compound statements

(i)   All rational numbers are real and complex.

(ii)   All real numbers are rationals or irrationals.

(iii) $x = 2$ and $x = 3$ are roots of the Quadratic equation $x^2 - 5x + 6 = 0$.

(iv)   A triangle has either 3-sides or 4-sides.

(v)   35 is a prime number or a composite number.

(vi)   All prime integers are either even or odd.

(vii) $|x|$ is equal to either x or $-x$.

(viii)   6 is divisible by 2 and 3.

7.   Rewrite each of the following statements in the form of conditional statements

(i)   The square of an odd number is odd.

(ii)   You will get a sweet dish after the dinner.

(iii)   You will fail, if you will not study.

(iv)   The unit digit of an integer is 0 or 5 if it is divisible by 5.

(v)   The square of a prime number is not prime.

(vi) $2b = a + c$, if a, b and c are in A.P.

8.   Form the biconditional statement $p \leftrightarrow q$, where

(i) p : The unit digit of an integer is zero.

    q : It is divisible by 5.

(ii) p : A natural number n is odd.

    q : Natural number n is not divisible by 2.

(iii) p : A triangle is an equilateral triangle.

    q : All three sides of a triangle are equal.

9.   Write down the contrapositive of the following statements:

(i)   If $x = y$ and $y = 3$, then $x = 3$.

(ii)   If n is a natural number, then n is an integer.

(iii)   If all three sides of a triangle are equal, then the triangle is equilateral.

(iv)   If x and y are negative integers, then xy is positive.

(v)   If natural number n is divisible by 6, then n is divisible by 2 and 3.

(vi)   If it snows, then the weather will be cold.

(vii)   If x is a real number such that $0 < x < 1$, then $x^2 < 1$.

10.   Write down the converse of following statements :

(i)   If a rectangle 'R' is a square, then R is a rhombus.

(ii)   If today is Monday, then tomorrow is Tuesday.

(iii)   If you go to Agra, then you must visit Taj Mahal.

(iv)   If the sum of squares of two sides of a triangle is equal to the square of third side of a triangle, then the triangle is right angled.

(v)   If all three angles of a triangle are equal, then the triangle is equilateral.

(vi)   If $x : y = 3 : 2$, then $2x = 3y$.

(vii)   If S is a cyclic quadrilateral, then the opposite angles of S are supplementary.

(viii)   If x is zero, then x is neither positive nor negative.

(ix)   If two triangles are similar, then the ratio of their corresponding sides are equal.

11. Identify the Quantifiers in the following statements.
    (i) There exists a triangle which is not equilateral.
    (ii) For all real numbers x and y, $xy = yx$.
    (iii) There exists a real number which is not a rational number.
    (iv) For every natural number x, $x + 1$ is also a natural number.
    (v) For all real numbers x with $x > 3$, x^2 is greater than 9.
    (vi) There exists a triangle which is not an isosceles triangle.
    (vii) For all negative integers x, x^3 is also a negative integers.
    (viii) There exists a statement in above statements which is not true.
    (ix) There exists a even prime number other than 2.
    (x) There exists a real number x such that $x^2 + 1 = 0$.

12. Prove by direct method that for any integer 'n', $n^3 - n$ is always even.
    [**Hint:** Two cases (i) n is even, (ii) n is odd.]

13. Check the validity of the following statement.
    (i) $p : 125$ is divisible by 5 and 7.
    (ii) $q : 131$ is a multiple of 3 or 11.

14. Prove the following statement by contradication method.
    p : The sum of an irrational number and a rational number is irrational.

15. Prove by direct method that for any real numbers x, y if $x = y$, then $x^2 = y^2$.

16. Using contrapositive method prove that if n^2 is an even integer, then n is also an even integers.